

Five-dimensional Supergravity Dual of a -Maximization

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abstract

We study the five-dimensional supergravity dual of the a -maximization under $\text{AdS}_5/\text{CFT}_4$ duality. We firstly show that the a -maximization is mapped to the attractor equation in five-dimensional gauged supergravity, and that the trial a -function is the inverse cube of the superpotential of the five-dimensional theory.

There is also a version of a -maximization in which one extremizes over Lagrange multipliers enforcing the anomaly-free condition of the R-symmetry. We identify the supergravity dual of this procedure, and show how the Lagrange multipliers appearing in the supergravity description naturally correspond to the gauge coupling of the superconformal field theory.

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1 Introduction

Conformal field theories (CFTs) are fascinating subjects in physics. Often they are strongly coupled, which means the application of naïve perturbation theory will be troublesome. However, CFTs have more symmetries in addition to the usual Poincaré symmetry. Two-dimensional CFTs, in particular, have an infinite number of symmetry generators, and the symmetry determines many of the physical properties. One can predict anomalous dimensions of various operators, and they agree with those of critical phenomena realized in nature.

CFTs in more than two dimensions remain more elusive. Supersymmetric conformal field theories (SCFTs) are somewhat better understood, since the algebra relates the scaling dimension to the R-charge. However, the precise understanding was limited, until quite recently, to the models with no global $U(1)$ symmetries other than the R-symmetry. In a seminal paper [1], Intriligator and Wecht showed how to proceed when there are several anomaly free R-currents. It is found that the correct R-symmetry can be found by maximizing the so-called trial a -function. This allowed various detailed study of previously unexplored SCFTs and flows between them; see for example [2, 3]. a -maximization has also found some phenomenological application [4].

Another way to approach the study of four dimensional CFT is the use of Anti-de Sitter (AdS)/CFT duality [5]. Indeed, using the prescription in [6, 7], any phenomena in CFT can be in principle mapped to those in the gravitational theory on AdS_5 . Many candidates were constructed by using D3 brane probing a Calabi-Yau cone with base X_5 . It predicts the duality between quiver gauge theories on the worldvolume of the brane and the type IIB string on $AdS_5 \times X_5$. One of the recent remarkable developments is that the gauge theory probing the tip of the toric Calabi-Yau cone can be constructed and analyzed by using the a -maximization (see for example [8]). Another is that we saw a great advancement in our understanding of the geometry of X_5 [9]. It has been discussed that the minimization of the volume of X_5 and the maximization of a always agree [10].

Much of the properties of the AdS side, however, can be studied without recourse to the machinery of string theory. We can consider the gravitational theory on AdS_5 by itself. The SCFT has at least eight supercharges, which means the corresponding theory on AdS_5 is a five dimensional supergravity with the same number of supercharges. It is known that such theories have a quite rigid structure which should be reflected on the properties of SCFTs. The analysis of this relation is performed for the maximal supergravity in [11] and for theories with sixteen and eight supersymmetries for example in [12–14]. We think that it is worthwhile to pursue this direction further, by utilizing the lessons from the recent advancement in understanding the SCFT in four dimensions. It will enable us to use the intuition in one side to explore the other in the AdS/CFT duality.

As the first step, we study the $\mathcal{N} = 2$ supergravity dual of the a -maximization of $\mathcal{N} = 1$ SCFT. We will find that the maximization procedure is mapped precisely to the attractor equation in supergravity. We will also find the dual of the a -maximization with the Lagrange multipliers introduced in [15, 16]. In that framework, anomalous and non-anomalous global symmetries are treated on the same footing, and the anomaly-free condition for the

superconformal R-symmetry is imposed by the Lagrange multipliers. It is proposed that these multipliers can be thought of as the coupling constants of the gauge fields. We will see that indeed there is a natural identification of the Lagrange multipliers with the coupling constants from the supergravity point of view.

The organization of the paper is as follows: we will first review in section 2 relevant aspects of the gauged supergravity in five dimensions [17,18]. The discussions will be brief and used mainly to fix the notations. Then in section 3, we show how the a -maximization is mapped to the attractor equation in supergravity. We also study the supergravity dual of the space of marginal deformations in SCFT. In section 4, we study the a -function with Lagrange multipliers in the supergravity side. We conclude in section 5 with some discussions and future prospects.

Note Added: An interesting paper [19] with some overlapping material with ours appeared on the eprint archive a week after we posted ours there.

2 Gauged supergravity in five dimensions

Let us recall the structure of gauged supergravity in five dimensions. The minimum number of supersymmetry generators is eight, and we concentrate on this case. It is called $\mathcal{N} = 2$ supergravity in the supergravity literature. There are several kinds of supermultiplet, and we restrict our attention to the gravity multiplet, vector multiplets and hypermultiplets. We restrict attention to the Abelian vector multiplets for brevity. Non-abelian gauge fields can be incorporated without much effort. We follow the conventions in [18].

2.1 Structure of scalar manifolds

Let us first discuss the scalars of the vector multiplets. Let us denote the number of vector fields by n_V . Then, there are $n_V - 1$ real scalar fields in the theory. When one compactifies one dimension, it will give $\mathcal{N} = 2$ supergravity in four dimensions. As such, the structure of the scalar manifold is determined by a unique function \mathcal{F} . A peculiarity in five dimension is that the third derivative of \mathcal{F} governs the Chern-Simons coupling and this fact fixes \mathcal{F} to be cubic,

$$\mathcal{F} = c_{IJK} h^I h^J h^K, \quad (1)$$

where h^I , ($I = 1, 2, \dots, n_V$), are the special coordinates. The scalar manifold M_V for the vector multiplet is given by the real $n_V - 1$ dimensional hypersurface defined by the constraint $\mathcal{F} = 1$ in the space of h^I . These manifolds are known as the very special manifolds [20]. Let ϕ^x , ($x = 1, 2, \dots, n_V - 1$), parametrize the manifold. It is useful to introduce the following quantities:

$$h_I \equiv c_{IJK} h^J h^K, \quad g_{xy} \equiv -3c_{IJK} h^I_{,x} h^J_{,y} h^K, \quad a_{IJ} \equiv h_I h_J + \frac{3}{2} g^{xy} h_{I,x} h_{J,y}. \quad (2)$$

We will raise and lower the indices I, J, \dots by using a_{IJ} .

Let us turn to the hypermultiplets. The manifold M_H of the hyperscalars is a quaternionic manifold of real dimension $4n_H$, which means that its holonomy is contained in $Sp(n_H) \times Sp(1)$ [21]. Let q^X , ($X = 1, \dots, 4n_H$), parametrize the manifold. We will introduce the vielbein f_{iA}^X where $i = 1, 2$ and $A = 1, \dots, 2n_H$ are the indices for $Sp(1)$ and $Sp(n_H)$ respectively. We normalize f_{iA}^X so that $f_{iAX} f_Y^i = g_{XY}$. Supersymmetry fixes the $Sp(1)$ part of the curvature [21] so that it is proportional to the triplet of almost complex structures:

$$R_{XYij} = -(f_{XiC} f_{Yj}^C - f_{YiC} f_{Xj}^C). \quad (3)$$

We trade the symmetric combination of two indices $\{ij\}$ for an index $r = 1, 2, 3$ by using the Pauli matrices, that is,

$$T_{ij} = \sigma_{ij}^r T_r \quad (4)$$

for any tensors. The $Sp(1)$ curvature R_{XY}^r satisfies the relation

$$R_{XY}^r R_{XZ}^s = \frac{1}{4} \delta^{rs} g_{YZ} + \frac{1}{2} \epsilon^{rst} R_{YZ}^t. \quad (5)$$

The kinetic terms for boson fields are then given by

$$\begin{aligned} e^{-1} \mathcal{L}_{\text{kin, boson}} = & -\frac{1}{2} R - \frac{1}{2} g_{xy} \partial_\mu \phi^x \partial^\mu \phi^y - \frac{1}{2} g_{XY} \partial_\mu q^X \partial^\mu q^Y \\ & - \frac{1}{4} a_{IJ} F_{\mu\nu}^I F^{J\mu\nu} + \frac{1}{6\sqrt{6}} e^{-1} c_{IJK} \epsilon^{\mu\nu\rho\sigma\tau} A_\mu^I F_{\nu\rho}^J F_{\sigma\tau}^K \end{aligned} \quad (6)$$

where R is the scalar curvature and e is the determinant of the fünfbein.

2.2 Gauging and the Potential

We need to introduce a scalar potential to get the AdS_5 vacuum. The structure of the potential is extremely restricted by the high degree of supersymmetries, and it must be accompanied by the gauging of the scalars. It also modifies the supersymmetry transformation.

We will take isometries K_I^X on M_H to covariantize the derivative

$$\partial_\mu q^X \rightarrow \partial_\mu q^X + A_\mu^I K_I^X. \quad (7)$$

The isometries can be expressed by the relation

$$K_I^X R_{XY}^{ij} = D_Y P_I^{ij}, \quad (8)$$

using the Killing potential P_I^{ij} . This is required by the consistency of the gauging with the supersymmetry.

The Killing potential P_I^{ij} appears in the Lagrangian. It gives the scalar potential as

$$V = \frac{3}{2} g^{xy} \partial_x P^{ij} \partial_y P_{ij} + \frac{1}{2} g^{XY} D_X P^{ij} D_Y P_{ij} - 2 P^{ij} P_{ij} \quad (9)$$

where $P_{ij} \equiv h^I P_{Iij}$.

It also appears in the covariant derivative of the gravitino ψ_μ^i ,

$$D_\nu \psi_\mu^i = \partial_\nu \psi_\mu^i + A_\nu^I P_{jI}^i \psi_\mu^j + \dots \quad (10)$$

P_{jI}^i enters in the covariant derivative of the gaugino as well.

It appears also in the supersymmetry transformation laws:

$$\delta_\epsilon \psi_\mu^i = D_\mu \epsilon^i + \frac{i}{\sqrt{6}} \gamma_\mu \epsilon^i P_{ij} + \dots, \quad (11)$$

$$\delta_\epsilon \phi^x = \frac{i}{2} \bar{\epsilon}^i \lambda_i^x, \quad (12)$$

$$\delta_\epsilon \lambda_x^i = -\epsilon_j \sqrt{\frac{2}{3}} \partial_x P^{ij} + \dots \quad (13)$$

$$\delta_\epsilon q^X = -i \bar{\epsilon}_i f^{XiA} \zeta_A, \quad (14)$$

$$\delta_\epsilon \zeta_A = \frac{\sqrt{6}}{4} \epsilon^i f_{XiA} K_I^X h^I + \dots \quad (15)$$

where λ_x and ζ_A are the gaugino and the hyperino, respectively.

Let us discuss a bit more about the isometry of the hyperscalars. The relation (8) can be solved to give P in terms of K as follows:

$$2n_H P_I^{ij} = D_X K_I^Y R_Y^{Xij}. \quad (16)$$

Consider a point on M_H so that $K_I^X = 0$, around which one can expand

$$K_I^X = Q_{IY}^X q^Y + \mathcal{O}(q^2). \quad (17)$$

Comparing with (7), we see that Q_{IY}^X determines the charge of the hypermultiplets. Then P_I^{ij} at the point is given by

$$P_I^{ij} = \frac{1}{2n_H} Q_{IY}^X R_X^{Yij}. \quad (18)$$

It means that P_I^{ij} is the $Sp(1)$ part of the charges Q_{IY}^X . Assuming Q s to be rational, P_I^{ij} at the point is also rational.

After this review, we can now move on to the study of the duality between $d = 4$ $\mathcal{N} = 1$ SCFT and the $d = 5$ $\mathcal{N} = 2$ gauged supergravity.

3 Dual of a -maximization

3.1 Brief review of a -maximization

Let us consider an $\mathcal{N} = 1$ SCFT in four dimensions. Let us denote by J_I^μ , ($I = 1, 2, \dots, n_V$), the currents of non-anomalous global symmetries of the theory and by Q_I corresponding

charges. We demand the charges Q_I to be integers, so that J_I can be coupled to external $U(1)^{nv}$ connections as follows:

$$S \rightarrow S + J_I^\mu A_\mu^I + \dots \quad (19)$$

Some of them may rotate the supercoordinates θ_α . Let the charges of θ_α under the global symmetry be given by \hat{P}_I :

$$\theta_\alpha \longrightarrow e^{i\phi^I Q_I} \theta_\alpha = e^{i\phi^I \hat{P}_I} \theta_\alpha \quad (20)$$

We will call a global symmetry $t^I J_I$ which commutes with θ_α a flavor symmetry. The condition is given by

$$t^I \hat{P}_I = 0. \quad (21)$$

Global symmetries, even if they are non-anomalous, may have chiral anomalies among them. This can be expressed by saying that the gauge transformation of the external $U(1)^N$ gauge field will have gauge anomaly described through the descent construction by the anomaly polynomial

$$\frac{1}{24\pi^2} \hat{c}_{IJK} F^I F^J F^K \quad (22)$$

where $F^I = F_{\mu\nu}^I dx^\mu \wedge dx^\nu / 2$ is the curvature two-form of the I -th external $U(1)$ gauge field. The constants \hat{c}_{IJK} are given by

$$\hat{c}_{IJK} = \text{tr } Q_I Q_J Q_K \quad (23)$$

where the trace is over the labels of the Weyl fermions of the theory. There may be gravitational anomaly given by

$$\hat{c}_I F^I \text{tr } R R \quad (24)$$

where R is the curvature two-form of the external metric.

A particular combination of global symmetries appears in the anticommutator of the supertranslation Q_α and the special superconformal transformation S^α :

$$\{Q_\alpha, S^\alpha\} \sim \tilde{s}^I Q_I. \quad (25)$$

We normalize \tilde{s}^I so that the charge of θ_α under $\tilde{s}^I Q_I$ be one, that is, $\tilde{s}^I \hat{P}_I = 1$. We denote the superconformal R-symmetry by $R_{SC} = \tilde{s}^I Q_I$.

R_{SC} can be used to uncover many physical properties of the theory considered. One is that the scalar chiral primary will have dimension Δ given by $\frac{3}{2} R_{SC}$. Another relation is with the central charges of the theory. In four dimensions, there are two of them, a and c , which are defined as the coefficients in front of the Euler density and the square of the Weyl tensor in the trace anomaly of the theory. They are expressible in terms of the superconformal R-symmetry as follows [22]:

$$a = \frac{3}{32} (3 \text{tr } R_{SC}^3 - \text{tr } R_{SC}), \quad c = \frac{1}{32} (9 \text{tr } R_{SC}^3 - 5 \text{tr } R_{SC}). \quad (26)$$

Suppose that some high-energy description of the (possible) SCFT is given. One can identify the non-anomalous symmetry and can calculate \hat{c}_{IJK} by using 't Hooft's anomaly matching. The charges \hat{P}_I of θ_α under J_I will also be easily given. Then, the basic problem is the identification of the superconformal R-symmetry $R_{SC} = \tilde{s}^I Q_I$.

Here comes the brilliant idea of Intriligator and Wecht [1]. Let $Q_F = t^I Q_I$ be a flavor symmetry, i.e. $t^I \hat{P}_I = 0$. They showed that the triangle diagram with one Q_F and two R_{SC} insertions can be mapped, by using the superconformal transformation, to the triangle diagram with Q_F and two energy-momentum tensor insertions. The precise coefficient was calculated to give the relation

$$9 \operatorname{tr} Q_F R_{SC} R_{SC} = \operatorname{tr} Q_F. \quad (27)$$

Another requirement is the negative definiteness

$$\operatorname{tr} Q_F Q_F R_{SC} < 0. \quad (28)$$

Let us introduce the trial a -function $a(s)$ for a trial R-charge $R(s) = s^\Lambda Q_\Lambda$ to be

$$a(s) = \frac{3}{32} (3 \operatorname{tr} R(s)^3 - \operatorname{tr} R(s)). \quad (29)$$

The conditions (27), (28) mean that $a(s)$ is locally maximized at the point $s^I = \tilde{s}^I$, where the trial R-charge becomes the superconformal R-charge R_{SC} . It is understood that s is constrained so that the charge of θ_α under $R(s)$ is one.

3.2 Supergravity dual of a -maximization

We would like to see how the a -maximization is translated under the AdS/CFT duality to the supergravity description. Let us first recall the prescription of AdS/CFT correspondence [6, 7]. For a current J_I^μ in the CFT side, we introduce a gauge field A_μ^I in the bulk which couples to the current at the boundary with the interaction $\int d^4x J_I^\mu A_\mu^I + \dots$. There are chiral anomalies among global symmetries generated by J_I , which translates to the fact that when the global symmetry is gauged, the partition function depends on the gauge, see (22). This can be reproduced through the bulk Chern-Simons coupling

$$\frac{1}{24\pi^2} \int \hat{c}_{IJK} A^I \wedge F^J \wedge F^K. \quad (30)$$

It is gauge invariant on a manifold without boundary, but it causes the partition function to vary appropriately on a manifold with boundary. This mechanism is the supergravity realization of the descent construction of consistent anomalies in the AdS/CFT correspondence [7]. Thus, we can identify the constants \hat{c}_{IJK} in the CFT side (23) and the constants c_{IJK} in the AdS side (6):

$$c_{IJK} = \frac{\sqrt{6}}{16\pi^2} \hat{c}_{IJK}. \quad (31)$$

Just in the same way, the chiral anomaly for the global symmetry–gravity–gravity triangle diagram is reproduced by the coupling

$$\int \hat{c}_I A^I \wedge \text{tr } R \wedge R \quad (32)$$

where R is the curvature two-form of the metric. It is, however, a higher derivative effect in the AdS side [23] so we neglect them in the rest of the paper. We hope to revisit the issue in a later publication. This means on the CFT side that we restrict attention to theories in which the chiral anomaly concerning gravity is much smaller than the chiral anomaly among three $U(1)$ symmetries.

For the rest of the section let us assume that the hyperscalar is at the point where $K_I^X = 0$ and concentrate on the behavior of the vector multiplets. Let us denote the charges of the hypermultiplet and the Killing potential by Q_{IY}^X and $P_I^{ij} = Q_{IY}^X R_X^{Yij}/(2n_H)$, respectively, as in (17). The global structure of M_H does not concern us¹. The global structure will be important if we study the flow between two supersymmetric vacua [24].

In order for the four-dimensional theory to be superconformal, the five-dimensional bulk should be AdS, and there should be eight covariantly constant spinors. To achieve this, we need to set the gaugini variation (13) to be zero,

$$P_I^r \tilde{h}_{,x}^I = 0. \quad (33)$$

We denoted the value of the quantity at the AdS vacuum by adding a tilde. This condition says that the three vectors P_I^r , $r = 1, 2, 3$, is perpendicular to the $n_V - 1$ row vectors $\tilde{h}_{,x}^I$ as vectors with n_V columns, which in turn means that P_I^r are parallel. Thus we can use the $Sp(1)$ global R-symmetry to set $P_I^r = \delta^{3r} P_I$ for some constants P_I . Then the equation (33) reduces to

$$P_I \tilde{h}_{,x}^I = 0. \quad (34)$$

This is an extremization condition for the superpotential $P \equiv P_I h^I$.

Now one can determine the commutation relation among the global symmetries which are respected by the vacuum. They are the isometries of AdS_5 , eight supercharges, and n_V global $U(1)$ symmetries from the gauge fields A_μ^I . From the covariant derivative of the gravitino (10), one finds that supercharges have charge $\pm P_I$ under the I -th global $U(1)$. Thus we can identify the quantity P_I introduced above and the quantity \hat{P}_I in the SCFT side which was introduced in (20). We will not distinguish P_I and \hat{P}_I in the following.

We have found the mapping under AdS/CFT duality of the basic constants \hat{c}_{IJK} and \hat{P}_I in the SCFT and c_{IJK} and P_I in the supergravity. Now we can study how the a -maximization is translated on the gravity side. Let us resume the study of the implication

¹For example, one can think of M_H as one of the Wolf spaces such as $Sp(n_H, 1)/Sp(n_H) \times Sp(1)$, and think of the Killing vectors as induced by the subgroup of the denominator $Sp(n_H) \times Sp(1)$. However, only the local properties of the metric near the zero of the Killing vectors are relevant, as long as we restrict our attention to the charges and the mass squared of the scalars as we will see below. Another thing one should notice is that the hyperscalar contains the dilaton, when the five-dimensional supergravity arises as the compactification of the type IIB string theory. It means that in general there are corrections to the metric of the hyperscalars. Hence the final metric will not be as simple as the one for the Wolf spaces.

of the condition (34) and recall the constraint $c_{IJK}h^I h^J h^K = 1$, which implies that $h_I h^I_{,x} = c_{IJK}h^I h^J h^K_{,x} = 0$. Thus h_I also is perpendicular to $n_V - 1$ vectors $h^I_{,x}$, from which we deduce that

$$\tilde{h}_I = c_{IJK}\tilde{h}^J\tilde{h}^K \propto P_I. \quad (35)$$

This is the attractor equation in the five-dimensional gauged supergravity [25].

Let us now identify the superconformal R-symmetry $R_{SC} = \tilde{s}^I Q_I$. From the supersymmetry transformation law for the hypermultiplets (14), (15), we can calculate the anticommutator of the supercharges acting on the hyperscalars. The result is

$$\{\delta_\epsilon, \delta_{\epsilon'}\}q^X = -i\frac{\sqrt{6}}{4}(\bar{\epsilon}\epsilon')\tilde{h}^I K_I^X + \dots, \quad (36)$$

from which we deduce that the anticommutator of the supercharges contains a $U(1)$ rotation $\propto \tilde{h}^I Q_I$. This $U(1)$ symmetry is identified under the AdS/CFT duality with the $U(1)_R$ symmetry in the superconformal algebra (25). Thus we find

$$\tilde{s}^I = t\tilde{h}^I. \quad (37)$$

where t is some proportionality constant. Let us next fix t . The gauge transformation law for the gravitino (10) signifies that the superconformal R-charge of the gravitino is $\tilde{s}^I P_I$. Considering that the superconformal R-symmetry is defined to rotate the gravitino by charge one, we need $\tilde{s}^I P_I = 1$. Thus we get

$$\tilde{s}^I = \tilde{h}^I / \tilde{P}, \quad (38)$$

where $\tilde{P} = \tilde{h}^I P_I$. Recall that a flavor symmetry $t^I Q_I$ satisfies $P_I t^I = 0$, see (21). Plugging this into the attractor equation (35), we obtain

$$c_{IJK}\tilde{h}^J\tilde{h}^K t^I \propto P_I t^I = 0. \quad (39)$$

This is precisely the condition (27) for theories with no chiral anomaly concerning gravity.

The other equation (28) is, by using (2), translated to the positivity of the metric of the scalar manifold [26]. To see this, let us recall that the $n_V - 1$ vectors $\tilde{h}^I_{,x}$ spans the vector space F defined by the condition

$$F = \{t^I \mid P_I t^I = 0\}. \quad (40)$$

Thus the positivity of the matrices $-\hat{c}_{IJK}\tilde{s}^I$ acting on F , equation (28), is translated to the positivity of the matrix $-\hat{c}_{IJK}\tilde{s}^I\tilde{h}^J_{,x}\tilde{h}^K_{,y}$, which is precisely the metric (2) of the vector multiplet scalars.

The maximization of the trial a -function $a(s^I)$ and the extremization of the superpotential $P = P(h^I)$ can be associated more explicitly. Let us generalize the relation (38) and relate the parameter for the trial R-symmetry $s^I Q_I$ and the value of the special coordinates h^I by the formula $s^I = h^I / (P_J h^J)$. Then, we have

$$a(s) \propto c_{IJK}s^I s^J s^K = \frac{c_{IJK}h^I h^J h^K}{(P_I h^I)^3} = (P_I h^I)^{-3}. \quad (41)$$

Thus, the trial a -function of the SCFT is precisely the inverse cube of the superpotential. Now it is trivial to see that the minimization of a is the maximization of P !

Let us carry out another consistency check. It is known [27–29] that, in a five-dimensional gravitational theory with the action

$$S = \frac{1}{2} \int d^5x \sqrt{g} (R + 12\Lambda + \dots), \quad (42)$$

the central charge a is given by

$$a = \pi^2 \Lambda^{-3/2}. \quad (43)$$

At the AdS vacuum, the negative of the vacuum energy is given from the potential (9) so that

$$6\Lambda = -V = 4(P_I \tilde{h}^I)^2. \quad (44)$$

Plugging the relation (38) and (44) into (43), we get

$$a = \pi^2 \left(\frac{3}{2} \right)^{3/2} c_{IJK} \tilde{s}^I \tilde{s}^J \tilde{s}^K = \frac{9}{32} \hat{c}_{IJK} \tilde{s}^I \tilde{s}^J \tilde{s}^K. \quad (45)$$

This agrees with the result from the field theory (26).

3.3 Mass squared of the vector multiplet scalar

The result presented in this and the next subsections is not new, see for example [13]. It is a preparation for section 3.5 and section 4.

We would like to study next the behavior of scalars in the vector multiplet around the vacuum \tilde{h}^I . We can calculate the second derivative of the potential there by using the special geometry relation

$$h^I_{,x;y} = \frac{2}{3} h^I g_{xy} + T_{xyz} h^I_w g^{zw} \quad (46)$$

where T_{xyz} is a certain completely symmetric tensor on M_V . Then,

$$D_x \partial_y V \big|_{h^I = \tilde{h}^I} = -g_{xy} \frac{8}{3} \tilde{P}^2 \quad (47)$$

Thus, the mass squared for all the scalar fields is negative with $m^2 = -4\Lambda$. Recall the classic relation

$$m^2/\Lambda = \Delta(\Delta - 4) \quad (48)$$

between the mass m in the AdS and the dimension Δ in the CFT. Then we have $\Delta = 2$ for all the $N_V - 1$ scalar fields, which barely saturates the Breitenlohner–Freedman bound [30] and thus the system is stable [31]. It is also easy to see that they have no R-charges. This is as it should be, because the scalar component of a vector multiplet corresponds to the lowest component of the current superfield, whose dimension is protected and whose R-charge is zero.

3.4 Dual of Scalar Chiral Primaries in SCFT

An important kind of multiplets in the $d = 4$ $\mathcal{N} = 1$ SCFT is the chiral multiplet, whose lowest component is a complex scalar we denote by \mathcal{O} . From the superconformal algebra the dimension and the R-charge is related through $\Delta(\mathcal{O}) = 3R(\mathcal{O})/2$. We would like to identify its supergravity dual. A natural candidate will be a hypermultiplet, which comes in quartets of real scalars. Chiral primaries, however, comes in pairs of real scalars. Naïvely there is twice the number of freedom in supergravity. We will see shortly below that the extra two degree of freedom corresponds to the F-component of the superfield \mathcal{O} .

Consider n_H hypermultiplets q^X with charges Q_{IY}^X under I -th $U(1)$ symmetry, where $X, Y = 1, \dots, 4n_H$. The charges under the superconformal R symmetry are given by

$$Q_Y^X \equiv \tilde{s}^I Q_{IY}^X = \tilde{h}^I Q_{IY}^X / \tilde{P}. \quad (49)$$

First, we would like to study the eigenvalues of Q_Y^X . To facilitate the task, let us combine the $4n_H$ real scalars into $2n_H$ complex scalars by introducing some complex structure J_X^Y so that Q_Y^X is diagonal. The relation (18) between P^r and Q_Y^X means that J_X^Y is proportional to $R_X^{Yr} P_r$. Let us form J^\pm from the other two complex structures so that

$$[J, J^\pm] = \pm 2J^\pm. \quad (50)$$

Then we can calculate the commutation relations of Q_Y^X and three J 's with the results

$$[J, Q] = 0, \quad [Q, J^\pm] = \pm 2J^\pm. \quad (51)$$

This means that the $2n_H$ eigenvectors can be arranged in pairs q_{iA} with charges r_{iA} , ($i = 1, 2$ and $A = 1, \dots, n_H$), so that $r_{1A} = r_{2A} + 2$. We further abbreviate so that $r_A \equiv r_{1A}$. One can also check that the supersymmetry relates q_{1A} and q_{2A} .

The mass squared m_{iA}^2 for the scalar q_{iA} can then be read off from the second derivative of the scalar potential:

$$\begin{aligned} D_X \partial_Y V|_{h^I = \tilde{h}^I} &= g^{ZW} D_X D_Z P^{ij} D_Y D_W P_{ij} - 4P_{ij} D_X D_Y P^{ij} \\ &= \frac{3}{2} Q_X^Z Q_{YZ} \tilde{P}^2 - 4\tilde{P}^{ij} R_{XZ}^{ij} Q_Y^Z. \end{aligned} \quad (52)$$

Substituting the diagonalized form of the charge matrix, we obtain the masses of the hypermultiplets as follows:

$$m_{1A}^2 = \frac{3}{2} r_A \left(\frac{3}{2} r_A - 4 \right) \Lambda, \quad (53)$$

$$m_{2A}^2 = \left(\frac{3}{2} r_A + 1 \right) \left(\frac{3}{2} r_A - 3 \right) \Lambda. \quad (54)$$

Thus, the scalar which is dual to q_{1A} under AdS/CFT has dimension $3r_A/2$ and R-charge r_A , and the one for q_{2A} has dimension $3r_A/2 + 1$ and R-charge $r_A - 2$. This combination of dimensions and charges are precisely the ones for the lowest component and the F component of a chiral multiplet.

3.5 Dual of Marginal Deformations

Let us next discuss the supergravity dual of exactly marginal deformations in SCFT,

$$S \rightarrow S + \int d^4x d^2\theta \tau_i \mathcal{O}_i \quad (55)$$

where the superconformal R-charge of the operators \mathcal{O}_i should be two. As remarked in [32], τ_i form a manifold M_c which parametrize the finite deformation. M_c naturally has a complex structure on it, which comes from the fact that the superpotential terms in $d = 4$ $\mathcal{N} = 1$ field theories have natural holomorphic structure.

We should be able to identify M_c in the framework of supergravity. As found in the last subsection, infinitesimal deformations with chiral primaries correspond to the hypermultiplet scalars. That the R-charge of a chiral primary is two means that the mass squared of the corresponding hypermultiplet scalar is 0 and -3 , and we saw the deformation $\int d^2\theta \tau_i \mathcal{O}_i = \tau_i[\mathcal{O}_i]_F$ corresponds to the two real scalars of mass squared zero. This in turn signifies that, when there are n chiral primaries of R-charge two, the supergravity vacuum comes in families with $2n$ real parameters. Let us call it M_{sugra}^c . This should be the supergravity realization of M_c . From the supersymmetry transformation law, we see that

$$M_{\text{sugra}}^c = \{p \in M_H \mid \tilde{h}^I K_I^X(p) = 0\}. \quad (56)$$

One thing is not obvious, however. The hypermultiplet scalars form a quaternionic manifold, which is definitely not a complex manifold. It is because the almost complex structures of a quaternionic manifold is not closed, but *covariantly* closed. Fortunately, we can easily check that the submanifold M_{sugra}^c is a Kähler manifold as follows.

First define $K^X \equiv \tilde{h}^I K_I^X$ and $P^r \equiv \tilde{h}^I P_I^r$ for brevity. From the property of the Killing potential $D_X P^r = R_{XY}^r K^Y$, P^r is covariantly constant on M_{sugra}^c . In particular, $P \equiv |P^r|$ is a constant parameter. Thus, J_Y^X defined by

$$J_Y^X \equiv R_Y^r g^{XZ} P_r / P \quad (57)$$

is an almost complex structure. This J_Y^X is covariantly constant with respect to the metric g_{XY} restricted from M_H onto M_{sugra}^c , because every factor in (57) is covariantly constant. It tells us that the metric on M_{sugra}^c has $U(n)$ holonomy, which means that M_{sugra}^c is Kähler.

4 Dual of a -maximization with Lagrange multipliers

4.1 a -maximization in the presence of anomalous currents

We saw in the previous section how the superconformal R-symmetry can be found as the combination of non-anomalous global currents, both from the SCFT and from the supergravity point of view. In [15, 16], Kutasov and collaborators incorporated anomalous global currents to the picture. It starts with the same trial a -function

$$a(s) = \frac{3}{32} (3 \text{tr}(s^I Q_I)^3 - \text{tr} s^I Q_I), \quad (58)$$

where Q_I now include all the global symmetries, anomalous or non-anomalous. Let us denote the a -th gauge fields by $F_{\mu\nu}^a$ and the Adler-Bell-Jackiw anomaly coefficient of the I -th global symmetry with a -th gauge field by m_I^a .

We need to impose the anomaly-free condition for each gauge group in the field theory considered. Thus we have to introduce the Lagrange multipliers λ_a and consider

$$a(s, \lambda) = a(s) + \lambda_a(m_I^a s^I). \quad (59)$$

We need to extremize it with respect to both s^I and λ_a . Define the function $a(\lambda)$ by first maximizing $a(s, \lambda)$ with respect to s^I , fixing λ_a . When $\lambda = 0$, the anomaly free condition is not imposed, and the a -function takes the value for the free field theory. This corresponds to zero gauge coupling. When λ attains the value $\tilde{\lambda}$ where $a(\lambda = \tilde{\lambda})$ is extremized, a becomes the true central charge of the SCFT. In [15] it was shown that a generically decreases along the flow of λ from zero to $\tilde{\lambda}$, suggesting that λ and the gauge coupling constants can be somehow identified. Indeed, there is a certain renormalization scheme where such identification is precise [16]. We would like to understand the supergravity dual of this procedure. We will see that in the framework of five-dimensional supergravity, the Lagrange multipliers and the gauge coupling constants can be naturally identified.

In [15, 16] Lagrange multipliers were also introduced for the condition that the superpotential should have R-charge two. Since the analysis for that case is basically identical, we concentrate on the interpretation of the Lagrange multipliers for the anomaly-free condition.

4.2 Supergravity Dual

First we need to study the supergravity dual for the anomalous global currents in SCFT. Let us first discuss without reference to supersymmetry. The conservation law is modified by the anomaly to be

$$\partial_\mu J^\mu = \mathcal{X} \quad (60)$$

for a suitable operator \mathcal{X} . An example of such current is a chiral $U(1)$ rotation which is broken by the instantons with $\mathcal{X} \propto \text{tr } F \wedge F$. Here F is the curvature of the gauge field which is not external, but is the constituent of the CFT considered.

In order to consider the gravity dual, we introduce external fields A_μ and ϕ defined on the AdS and the coupling $\int dx^4 A_\mu J^\mu + \phi \mathcal{X}$ on the boundary. We can see now [33] that if the bulk gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \epsilon$ is accompanied by the transformation $\phi \rightarrow \phi - \epsilon$, the prescription of AdS/CFT correspondence leads to the anomalous conservation law (60).

Let us now consider the effect of supersymmetry. Consider an anomalous flavor symmetry. The current J^μ is then incorporated into a current superfield J and the operator \mathcal{X} is the imaginary part of the F-component of the superfield $\mathcal{O} \propto \text{tr } W_\alpha W^\alpha$. The supersymmetry completion of (60) becomes

$$\bar{D}^2 J = \mathcal{O}. \quad (61)$$

This is the celebrated Konishi anomaly [34].

We saw that a current superfield in SCFT corresponds to a vector multiplet and that a chiral multiplet to a hypermultiplet. Thus, the Konishi anomaly is dual in the supergravity description to the Higgsing of a vector multiplet eating a hypermultiplet. After Higgsing, the multiplet is no longer short. Thus the dimension of the operators is not protected anymore. However supersymmetry relates the anomalous dimension of J and the anomalous dimension of \mathcal{O} [35].

Let us examine how the incorporation of these massive vector multiplets is reflected to the a -maximization. Consider the chiral operators $\mathcal{O}_a = \text{tr } W_a^\alpha W_{a\alpha}$ which yield kinetic terms for the a -th non-Abelian gauge fields, where $a = 1, \dots, n'_H$ label the factor of the gauge groups. We do not sum over a inside the trace. Define the isometries K_a^X so that, if the Konishi anomaly for the I -th current superfield is given by

$$\bar{D}^2 J_I \propto m_I^a \mathcal{O}_a, \quad (62)$$

the vector field A_μ^I in the five-dimensional supergravity gauges the direction $m_I^a K_a^X$. K_a^X is non-zero at the vacuum.

Other n''_H hypermultiplets which are not Higgsed are also charged under A_μ^I . We denote the Killing vectors for these by $K_{(0)I}^X$. We assume that this can be expanded as

$$K_{(0)I}^X = Q_{IY}^X q^Y + \mathcal{O}(q^2) \quad (63)$$

as before. The total gauging K_I^X appearing in the supergravity Lagrangian is given by

$$K_I^X = m_I^a K_a^X + K_{(0)I}^X, \quad (64)$$

and we denote the corresponding Killing potential as

$$P_I^r = m_I^a P_a^r + P_{(0)I}^r. \quad (65)$$

Let us study the condition for the AdS vacuum, which can be found by inspecting the hyperino and gaugino transformation laws. A convenient parametrization of the hyperscalars near the vacuum is given as follows: near the zero of $K_{(0)I}^X$, let it be linearly dependent on the scalars $q^{\hat{X}}$ where $\hat{X} = 1, \dots, 4n''_H$. We need $4n'_H$ coordinates in addition. n'_H of them are the gauge orbits along K_a^X . We can take P_a^r as the remaining $3n'_H$ of the coordinates. They are guaranteed to form good local coordinates because

$$D_X P_a^{ij} = R_{XY}^{ij} K_a^X \neq 0. \quad (66)$$

The hyperino transformation law gives the condition

$$\tilde{h}^I K_I^X = 0. \quad (67)$$

Its first consequence is that

$$\tilde{h}^I m_I^a = 0. \quad (68)$$

Recall the superconformal R-charge is proportional to $\tilde{h}^I Q_I$, see (38). This translates in the SCFT language to the fact that anomalous global currents do not participate in the superconformal R-charge. Assuming other linear combination of \tilde{h}^I is non-zero, we can see that $q^{\hat{X}} = 0$. However, hyperino variation alone does not fix P_a^r .

Next, let us turn to the gaugino variation

$$\tilde{h}_{,x}^I P_I^{ij} = 0. \quad (69)$$

Just as before, it says that the vectors with n_V elements $P_I^{r=1,2,3}$ are all parallel to h_I . Global $Sp(1)$ rotation can be used so that P_I^r is nonzero only for $r = 3$. Let us note that, from the relation (63), $P_{(0)I}^{r=1,2}$ is quadratic in the fields $q^{\hat{X}}$. Combining with the equation (65), we get $P_a^{r=1,2} = 0$. Thus, the remaining variables are $(n_V - 1)$ vector multiplet scalars and n'_H coordinates of the hyperscalar, $P_a^{r=3}$. Now define the superpotential to be the gravitino variation

$$P \equiv h^I P_{(0)I} + P_a^{r=3} m_I^a h^I \quad (70)$$

where the parameter is the vector multiplet scalars h^I with the constraint $c_{IJK} h^I h^J h^K = 1$ and the hypermultiplet scalars $P_a^{r=3}$. Extremization condition for those scalars yields precisely the conditions (67) and (69). Thus, the AdS vacua can be found by extremizing the superpotential P with respect to h^I and $P_a^{r=3}$. We can see that the scalars $P_a^{r=3}$ work as the Lagrange multipliers enforcing the condition $h^I m_I^a = 0$. Surprisingly, the Lagrange multipliers are physical fields in the supergravity side!

The way of introducing multipliers in the gauge theory (59) and in the supergravity (70) is not exactly the same. We know that however, when we want to extremize a quantity, say $a(x)$, with respect to x in the presence of the constraint $c(x) = 0$, it is immaterial whether we choose $a_1(x, \lambda) \equiv a(x) + \lambda c(x)$ or $a_2(x, \lambda) \equiv f^{-1}(f(a(x)) + \lambda c(x))$. The difference between (59) and (70) is of this form, thus of no physical relevance.

We have seen in section 3 that $(n_V - 1)$ vector multiplet scalars corresponds to the trial R-charge through the relation (38), and that the dual SCFT operator is the scalar in the current superfield. Then it is natural to ask the same question on the scalars $P_a^{r=3}$. It acted as the Lagrange multipliers in the superpotential extremalization. As a final exercise in this paper, let us identify the SCFT dual of the scalar $P_a^{r=3}$. The fact that $\tilde{h}^I P_I^r$ is nonzero only for $r = 3$ means that the superconformal R-symmetry is the $U(1)$ subgroup specified by σ^3 of the global $Sp(1)$ R-symmetry of the ungauged theory. This can be read off just as in the discussion in the section 3.2. This tells us that $P_a^{r=3}$ has zero superconformal R-charge, while $P_a^{r=1,2}$ has charge two.

We have already seen that the gauge orbit K_a^X corresponds to the topological density $\text{tr } F^a \wedge F^a$. Three real scalars P_a^r are its supersymmetric partners. From the discussions in section 3.4, we can infer that P_a^r correspond under AdS/CFT duality to the three operators

$$\text{tr } F_{\mu\nu}^a F_{\mu\nu}^a, \quad \text{Re tr } \lambda_\alpha^a \lambda^{a\alpha}, \quad \text{and} \quad \text{Im tr } \lambda_\alpha^a \lambda^{a\alpha}. \quad (71)$$

Comparing the R-charges, we find that $P_a^{r=1,2}$ are the dual for the gaugino bilinears and that $P_a^{r=3}$ corresponds to the kinetic term $\text{tr } F_{\mu\nu}^a F_{\mu\nu}^a$. Let us recall that the prescription of

AdS/CFT duality [6, 7] means that there is a boundary interaction

$$\int dx^4 P_a^{r=3} \text{tr} F_{\mu\nu}^a F_{\mu\nu}^a. \quad (72)$$

Thus, we found that the Lagrange multiplier $P_a^{r=3}$ corresponds precisely to the gauge coupling constant under AdS/CFT duality.

5 Conclusion and Discussions

In this paper, we showed how the a -maximization in the SCFT is mapped to the attractor equation in five-dimensional supergravity. We saw that the lowest derivative terms of the gravitational theory is determined by the very quantities \hat{c}_{IJK} and \hat{P}_I which enters the a -maximization. We also saw how it is related to the attractor equation thanks to the structure of the very special structure of the scalar manifold. Furthermore, we studied the supergravity dual of another version of the a -maximization, which is formulated as the extremization over the Lagrange multipliers enforcing the anomaly free condition. We showed that the Lagrange multipliers also appear in the supergravity description, and that they naturally corresponded to the gauge coupling constant. This agrees with the expectations from the analysis in four-dimensional field theory.

We would like to discuss about the possible directions of research. An immediate concern is the determination of c_{IJK} when the five-dimensional theory is obtained as the compactification of the type IIB superstring on a Sasaki-Einstein X_5 . For the CY compactification of M-theory, we can easily show that c_{IJK} is none other than the triple intersection product of the two-cycles in the CY. We could expect some kind of topological formula for c_{IJK} given the topology of X_5 , especially if X_5 is toric.

There also are several questions purely within the realm of correspondence between five-dimensional supergravities and four-dimensional SCFTs. First is the incorporation of $A^I \wedge \text{tr} R \wedge R$ term in the supergravity. This will require the construction of five-dimensional gauged supergravity with higher derivative terms. Second is the study of the dual of the Higgsing in the SCFT. SCFT can often be deformed by giving non-zero vacuum expectation values to chiral primaries, and the moduli of such deformations will be a Kähler cone. It will be extremely interesting if we can realize this Kähler cone from the scalar manifold of supergravity. It may be possible to phrase the result in [36] in some purely five-dimensional parlance. We would like to visit these issues in future publications.

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